Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 7 Solution

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Course Title: Mobile Communications
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Questions

1. Wireless systems suffer from multipath fading problem. Equalization can be used to mitigate this problem. Another important technique that works effectively in wireless systems is OFDM. The general idea is to increase the symbol or bit time so that it is large compared with the channel delay spread. To do this, we separate the original data stream into multiple parallel substreams and transmit the substreams via different carrier frequencies, creating parallel subchannels. This is called FDM. In such direct implementation, there are two new problems to solve: bandwidth inefficiency and complexity of the transceivers. The inefficient use of bandwidth is caused by the need of guard bands between adjacent subchannels. Bandwidth efficiency can be improved by utilizing orthogonality. The computational complexity of the transceivers is solved by the use of FFT and IFFT.

2. In this question, we will consider an OFDM system in discrete time. The channel is characterized by $\mathbf{h} = \begin{bmatrix} 2 & -1 \end{bmatrix}$. We would like to transmit a stream $\begin{bmatrix} 1 & -1 & 2 & 1 & -1 & 2 & 1 & 2 \end{bmatrix}$ of data across this channel using OFDM. For simplicity, we will assume that there is no noise. Let N = 4 be the length of each OFDM symbol.

    a. First note that because we have N = 4 but there are 8 numbers that we want to transmit; we will need two OFDM symbols; one for $\begin{bmatrix} 1 & -1 & 2 & 1 \end{bmatrix}$ and another one for $\begin{bmatrix} -1 & 2 & 1 & 2 \end{bmatrix}$.

    To apply IFFT to a (column) vector of length 4, simply multiply it by $\mathbf{\Psi}_4^{-1} = \dfrac{1}{N}\mathbf{\Psi}_4^{*}$. In class, we have already seen that

$$\Psi_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \psi_N^{-1} & \psi_N^{-2} & \cdots & \psi_N^{-(N-1)} \\ 1 & \psi_N^{-2} & \psi_N^{-4} & \cdots & \psi_N^{-2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \psi_N^{-(N-1)} & \psi_N^{-2(N-1)} & \cdots & \psi_N^{-(N-1)(N-1)} \end{bmatrix} \text{ where } \psi_N = e^{j\frac{2\pi}{N}}.$$

Here, with $N = 4$, we have $\Psi_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$ and $\Psi_4^{-1} = \dfrac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$.

So, the IFFT of $\begin{bmatrix} 1 & -1 & 2 & 1 \end{bmatrix}$ is $\dfrac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix} = \dfrac{1}{4}\begin{bmatrix} 3 \\ -1-2j \\ 3 \\ -1+2j \end{bmatrix}$ and

The IFFT of $\begin{bmatrix} -1 & 2 & 1 & 2 \end{bmatrix}$ is $\dfrac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & j \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 1 \\ 2 \end{bmatrix} = \dfrac{1}{4}\begin{bmatrix} 4 \\ -2 \\ -4 \\ -2 \end{bmatrix}$.

After scaling by $\sqrt{N}$ we have $\dfrac{1}{2}\begin{bmatrix} 3 \\ -1-2j \\ 3 \\ -1+2j \end{bmatrix}$ and $\dfrac{1}{2}\begin{bmatrix} 4 \\ -2 \\ -4 \\ -2 \end{bmatrix}$.

Now, to add cyclic prefixes, we note that the memory length of the channel is 2-1 = 1. So, the cyclic prefix length for each block is 1. This gives

$$\mathbf{x} = \dfrac{1}{2}\begin{bmatrix} -1+2j, & 3, & -1-2j, & 3, & -1+2j, & -2, & 4, & -2, & -4, & -2, \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{1}{2}+j, & \dfrac{3}{2}, & -\dfrac{1}{2}-j, & \dfrac{3}{2}, & -\dfrac{1}{2}+j, & -1, & 2, & -1, & -2, & -1 \end{bmatrix}.$$

b. $\mathbf{y} = \mathbf{x} * \mathbf{h}$. (Note that this is a regular convolution.) Here,

$$\mathbf{y} = \begin{bmatrix} -1+2j, & \dfrac{7}{2}-j, & -\dfrac{5}{2}-2j, & \dfrac{7}{2}+j, & -\dfrac{5}{2}+2j, & -\dfrac{3}{2}-j, & 5, & -4, & -3, & 0, & 1 \end{bmatrix}..$$

c. To find $\mathbf{H}$ which is the FFT of the zero-padded $\mathbf{h}$, we first need to think about how many 0s should be padded to $\mathbf{h}$. Because $N = 4$, we need to zero-pad $\mathbf{h}$ by two zeros to bring it to the same size as a data block for OFDM.

So, $\mathbf{H} = \boldsymbol{\Psi}_4 \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2+j \\ 3 \\ 2-j \end{bmatrix}.$

d. First we remove the "irrelevant parts" from $\mathbf{y}$:

$$\left[-1+2j, \quad \frac{7}{2}-j, \quad -\frac{5}{2}-2j, \quad \frac{7}{2}+j, \quad -\frac{5}{2}+2j, \quad -\frac{3}{2}-j, \quad 5, \quad -4, \quad -3, \quad 0, \quad 1 \right].$$

Note that the first two removals correspond to the locations of the cyclic prefixes. The last removal is the extra junk we get from doing regular convolution.

So, now, we have two received blocks:

$$\left[\frac{7}{2}-j, \quad -\frac{5}{2}-2j, \quad \frac{7}{2}+j, \quad -\frac{5}{2}+2j \right] \text{ and } \left[5, \quad -4, \quad -3, \quad 0 \right].$$

Note that, earlier, we added the cyclic prefixes so that these received blocks are the circular convolutions of (the inverse-transform of) our data (in time domain) with the (zero-padded) channel. This implies that, in the frequency domain, our data are simply multiplied element-by-element by $\mathbf{H}$.
So, to recover our data, we convert the received blocks back to frequency domain via FTT. This gives

$$\left[2 \quad -4-2j \quad 12 \quad 4-2j \right] \text{ and } \left[-2 \quad 8+4j \quad 6 \quad 8-4j \right].$$

Recall that, in the transmission part, we scale our block by $\sqrt{N}$. To cancel this scaling, we need to add another scaling of $1/\sqrt{N}$ which gives

$$\left[1, \quad -2-j, \quad 6, \quad 2-j \right] \text{ and } \left[-1, \quad 4+2j, \quad 3, \quad 4-2j \right].$$

Finally, the circular convolution (in time) for DFT tells us that in the frequency domain, this received blocks are simply the data blocks multiplied element-wise by $\mathbf{H}$. Therefore, to recover the data blocks, we divide the elements in the vectors above by the corresponding elements in $\mathbf{H}$ to get

$$\left[1 \quad -1 \quad 2 \quad 1 \right] \text{ and } \left[-1 \quad 2 \quad 1 \quad 2 \right].$$

Joining these blocks together, we then get back our original data stream

$$\left[1 \quad -1 \quad 2 \quad 1 \quad -1 \quad 2 \quad 1 \quad 2 \right].$$